Reports from the Trenches in the Math Wars

Comments of Fred Greenleaf, Professor of Mathematics, New York University, presented at a panel organized by the National Association of Scholars, 22 May 2004.

I would like to speak about the disparity between K-12 programs and the math needed for college success. This has always been a serious issue. Large numbers of high school graduates hope to enter college programs. Many NCTM-based K-12 math programs offer poor preparation for entry to college, or to succeed in college programs having significant mathematical components. Since NCTM-based programs (*Everyday Math*) have now become the mandated K-6 math curriculum in most NYC schools, and since these are the grades in which the foundations of later math learning must be laid, we expect this disparity to become far more dramatic in the near future.

Why is a University math professor mouthing off about K-12 math teaching?

When Elizabeth Carson first appeared at my office door six years ago, I had some concerns about K-12 math education, but had no idea idea just how bad the NCTM-based math programs being introduced in the City really were. Before then I had been responsible for several major curriculum development projects at NYU, many of which had to confront the effects of inadequate math backgrounds among entering students. Naturally, I was concerned about the K-12 programs that led to these difficulties, but found it hard to believe things were about to get much worse.

My own enlightenment began when I conducted numerous discussions and interviews with NYC math teachers, especially in NYC District 2 where NCTM-based programs were first introduced, to learn how these programs impact experienced math teachers. The results were disturbing; this unease turned to alarm as I systematically examined various NCTM-based programs TERC (K-5), CMP (6-8), ARISE (9-12), IMP (9-12), comparing them to the Singapore Math curriculum – a program whose students consistently lead the world in math ability.

In the past five years, my colleagues and I have become deeply concerned about these issues. It was clear to us that the NCTM programs, if they remained in place, were going to have a terrifically negative impact on the prospects of all students who aspire to college. Many experienced math teachers I interviewed complained that the NCTM-based courses tend to be "dumbed-down," with one 6^{th} grade teacher observing

> " I've been teaching math for a long time, and am struck by how much less math actually gets covered under these new programs, *compared to what got accomplished just a few years ago*."

The emphasis is mine; I doubt that students have suddenly gotten much worse, but the math programs certainly have!

The profound inequity of these programs should also be noted. Students whose parents are not affluent enough to afford extensive tutoring in the middle-

and high school years are simply out of luck. As one very experienced and competent 4th grade math teacher told me,

> "The only one who is going to benefit from these programs is Stanley Kaplan. For him they are a godsend."

What do students need at the college level?

In my experience the greatest single obstacle to success for students entering college – even in courses for non-math majors – is lack of PROFICIENCY in algebra and symbolic thinking. To elaborate, I would say that students need

- Fluent and automatic skills in algebra and symbolic thinking
- Basic concepts from geometry, up through trigonometry.
- Basic numeracy: confidence in dealing with decimals, ratios, proportions,etc.
- The ability to connect these skills with the real world – to handle verbal problems, translate them into mathematical form, and solve them.

The emphasis here is on "fluent and automatic" ability. If you have you have been through a curriculum that denigrates mastery of skills as "rote learning," or avoids symbolic methods in favor of drawing pictures, you will have real troubles at the college level. A quote from an article *In defense of rote learning*, by Ethan Aiken, seems appropriate here.[1] Making an analogy between learning to read and learning to use mathematics, he notes

> "Success at learning to read begins with being able to recognize letters instantly, without conscious effort."

The same can be said of the algebraic skills that underlie most mathematics.

Failure to develop these skills can prevent a student from gaining entry to college. For instance, in the CUNY system about 70% of all applicants fail the simple university-wide test of math skills, and are denied entry to the senior colleges; at best they are shunted into remedial programs at a local community college. Once accepted to college, they find that many career paths require sufficient math skills to pass courses in calculus and statistics. These careers include: premedical and bioscience programs, business school programs, programs in architecture, design, economics, environmental science, engineering, the physical science, and of course mathematics. Those without the necessary math skills are excluded.

[1] Aiken begins his article with a quote from Alfred North Whitehead: "It is a profoundly erroneous truism repeated by all copybooks, and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of operations which we can perform without thinking about them. Operations of thought are like cavalry charges in battle – they are strictly limited in number, they require fresh horses, and should only be made at decisive moments."

Many adherents to the NCTM philosophy delude themselves into thinking that the college level emphasis on calculus and symbolic manipulation skills is archaic and will soon wither away in this age of calculators – that all levels of math education will soon merge into a seamless K-16 curriculum, all based on their principles of "learning by discovery" and without any need for what they refer to as "rote learning."

It will not happen. In college level mathematics, the calculator is the *last* tool you reach for – after you have analyzed a verbal problem, cast it in mathematical form, and applied enough algebraic technique to reduce the solution to a simple arithmetic calculation. Those who hope that the advent of calculators that do algebra will eliminate the need to actually learn algebra may be disappointed to discover that you have to be quite proficient in algebra to make use of such systems.

Finally, practitioners of the mathematical arts know that you have to experience math at one or two levels beyond the level at which you will employ math in everyday practice, in order to do it well. Hence the emphasis on calculus as a training ground, say for architects who will never again compute an integral once they graduate. That is true – but they will have to be quite proficient at trigonometry and geometry, must have a highly developed number sense, and will have to speak fluently with experts in such highly mathematical areas as stress analysis, etc etc. The folks who designed their undergraduate program know this, and will continue to retain calculus-level mathematics requirements.

What is to be done?

Here are a few modest proposals.

1. *Acknowledge that the math curriculum actually matters.* California was one of the first states to adopt the newly developed NCTM-based programs, such as TERC (*Investigations in Number and Space*) in the elementary grades. It was also the first to decisively reject them, in part because of the soaring need for remediation among students entering the California system of colleges and universities. Beginning in 1997 the State reformulated its math standards for grades K-8, this time with strong input from practicing mathematicians as well as education specialists. These Standards spelled out a curriculum that focused on core topics, mastery of basic skills at each grade level, and a coherent curriculum that would lead students through basic algebra by the end of 8^{th} grade. *None* of the NCTM-based curricula made the state's appoved list of texts consistent with the goals of these standards. The NYC curriculum of choice, *Everyday Math*, has still not been approved in California although it has been revised since 2000 to downplay some of the NCTM-inspired excesses of the first edition.

A recent report by Wayne Bishop and William Hook[1] follows several cohorts of students through grades 2 – 5, comparing yearly statewide math scores of students whose schools adopted the programs recommended in the new Standards, with

[1] *Urban elementary schools in California show stunning improvement in SAT-9 scores over initial four year period of new Math Standards*, by Wayne Bishop, Department of Mathematics, California State University Los Angeles, and William Hook, Biology Department, University of Victoria, British Columbia; 12 pp, January 2004, to appear.

schools in other districts that insisted on a temporary dispensation from them. At every grade level, math scores rose statewide during the study period 1998 – 2002, partly due to state mandated reductions in class size. However, the rise in scores at schools adhering to the new standards dramatically surpassed that at schools which retained programs based on the NCTM philosophy. Furthermore, this was true for all demographic profiles.

One step in the right direction might be to acknowledge these results, and take close look a look at the California Standards and curricular materials that are aligned with those standards.

2. *The Role of College Math Departments.* Sherman Stein, in his 1996 book *Strength in Numbers*, reports that at his school, University of California at Davis, about half of all graduates from undergraduate math programs end up as high school math teachers. That sounds higher than the percentage at some schools I have known, but I am convinced that a large cadre of our undergraduate majors will enter this career path, and that this constituency will be even more important in the future.

Those graduates will play a crucial role in preparing students for college level work. My question is this: Are math departments paying sufficient attention to the *real* needs of this large constituency? Their needs are not the same as those of majors planning to go on to higher degrees, the group math departments have traditionally favored. It is time for Math Departments to step up to the plate.

3. *Alternative certification programs.* Within the next year or two, all candidates for New York State certification to teach a disciplinary subject at the middle or high school level will be required to have undergraduate major degree in that (or a closely related) subject, *meeting degree requirements specified by the disciplinary departments, rather than the School of Education in which they may be enrolled.*

As the revised New York State rules for subject certification come into full force, I have serious doubts that Schools of Education will be able to recruit enough students capable of completing a math (or science) major *and* the usual degree program in education. Indeed, owing to the present shortage of math teachers, someone who has completed an undergraduate math major could get temporary certification, and by completing two years of summer programs in education become fully certified.

I believe this trend should be strongly encouraged. Math training has, in my experience, been woefully neglected in many Schools of Education, or severely distorted in the name of ill-conceived educational ideologies. What might help is a re-invention of the Master of Arts in Teaching, as a program in which Math Departments and Schools of Education are equal participants.

I have seen may fine students of mathematics decide, often well after completing their undergraduate degrees, that their interests lay in teaching. Graduates who make this career change would be encouraged by the existence of brief, well-focused programs of this sort.

4. *A Role for Math Specialists.* I am less confident that the training of K-6 elementary teachers can be reformed. For openers, I modestly propose that all school system administrators responsible for math education policy should have

at least completed a minor in mathematics. That would exclude a good number of NYC administrators who have been mandating math curricula and dictating math teaching policy for the past several years.

Teaching K-6 math concepts to *real* K-6 students is a daunting task, one requiring deep knowledge of mathematics, great pedagogical skill – the ability to go at a topic five different ways if necessary – and enormous empathy for student who are groping their way toward understanding. What it takes is beautifully set forth in Liping Ma's book *Knowing and Teaching Elementary Mathematics*, which compares K-6 teaching methods in China with those in the US. Chinese teachers operate with the ultimate goal

> "Know how, and also know why"

They seek to have their students achieve confident proficiency as well as deep understanding – a sense of balance sorely lacking in many US programs. The Chinese model also acknowledges the extraordinary challenges of teaching math well at this level by assigning this task to cadres of dedicated math specialists who, working together, continually study math concepts and devise improved ways to present them in class.

It has always struck me as absurd that we expect K-6 teachers, with almost no training in math content, to be able to teach math well to learners at this stage. It is time to consider putting this crucial aspect of early education into the hands of well-paid specialists, fluent in math content as well as pedagogy, who can devote their full time to the teaching of this subject. Liping Ma's book is a blueprint for how this might be done.

Of all the things that might be done, I feel this is the most important. The K-6 period is where the foundation for all later work in mathematics is laid. If the foundation is flawed, all later efforts are compromised.